

Sznajd Complex Networks

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Abstract

The Sznajd cellular automata corresponds to one of the simplest and yet most interesting models of complex systems. While the traditional two-dimensional Sznajd model tends to a consensus state (pro or cons), the assignment of the contrary to the dominant opinion to some of its cells during the system evolution is known to provide stabilizing feedback implying the overall system state to oscillate around null magnetization. The current article presents a novel type of geographic complex network model whose connections follow an associated feedbacked Sznajd model, i.e. the Sznajd dynamics is run over the network edges. Only connections not exceeding a maximum Euclidean distance D are considered, and any two nodes within such a distance are randomly selected and, in case they are connected, all network nodes which are no further than D are connected to them. In case they are not connected, all nodes within that distance are disconnected from them. Pairs of nodes are then randomly selected and assigned to the contrary of the dominant connectivity. The topology of the complex networks obtained by such a simple growth scheme, which are typically characterized by patches of connected communities, is analyzed both at global and individual levels in terms of a set of hierarchical measurements introduced recently. A series of interesting properties are identified and discussed comparatively to random and scale-free models with the same number of nodes and similar connectivity.

1 Introduction

Introduced by Sznajd-Weron and her father Sznajd in 2000 [1], the Sznajd cellular automata represent one of the simplest and most interesting models of complex systems. Typically considered as a model of opinion formation, the Sznajd model is known to undergo phase transitions in specific circumstances [2]. Al-

though recent, such a model has motivated a whole series of investigations — the interested reader should refer to [3, 4] for updated surveys. While the traditional two-dimensional Sznajd model is known always to reach consensus (all pro or all cons) after its evolution, the assignment of the contrary of the dominant opinion to some nodes during the evolutionary dynamics is known [5] to provide a stabilizing effect in the sense that the system may now oscillate around null magnetization (i.e. balance of pros and cons). The two-dimensional spatial distribution of such states involves a series of interconnected patches of different sizes and shapes, which suggests the use of the Sznajd model for pattern formation studies.

Also introduced recently, complex networks have motivated their own focus of attention from the complex systems community [6, 7]. Representing a promising interface between two well-established areas, namely graph theory and statistical mechanics, this new area provides interesting possibilities for representation, characterization and simulation of systems with complex connectivity as well as the respective dynamics [7]. Initiated by the pioneering studies by Flory [8], Rapoport [9] and Erdős and Rényi [10], the area of complex networks has been catalysed by the more recent developments by Watts and Strogatz [11] and Barabási and collaborators [12]. A great part of the current interest in this area stems from scaling laws such as the power-law characteristic of the Barabási-Albert model, whose main importance lies in the enhanced probability of obtaining hubs. By concentrating the network connectivity, such hub nodes have been found to be of paramount importance for the network topology, dynamics, and resilience to attack [6]. Although complex networks are typically characterized and analyzed in terms of the topological measurements known as node degree and clustering coefficient, a set of hierarchical measurements, including respective extensions of these two concepts, was introduced recently [13, 14, 15] which allows a sub-

stantially more comprehensive characterization of the connectivity of the analyzed networks.

The current article brings together the two above areas of Sznajd models and complex networks. Although Sznajd dynamics has already been performed on such networks, we describe what is possibly the first study involving Sznajd dynamics over the network connectivity. More specifically, we suggest a novel geographic complex network model whose connections are defined as a consequence of the evolution of a respectively associated Sznajd model with contrary feedback. The node positions are distributed according to the Poisson density (i.e. the nodes are positioned inside the $L \times L$ square with uniform probability), and only pairs of nodes lying within Euclidean distance D one another are allowed to connect (see also [16, 17]) during the subsequent processing stage. While a model obeying such rules would exhibit regular features, in the sense that the node degrees of all nodes would be close to their overall average (small node degree dispersion), the suggested model involves a dynamical evolution of the connections by taking into account a respectively associated feedbacked Sznajd model which implies enhanced node degree dispersion as a consequence of border effects of the obtained patched communities. The initial network is obtained by assigning, with uniform probability p , connections to each pair of nodes which are no further apart than D . The growth procedure involves selecting, one at a time, a pair of nodes lying no further than D . In case these two nodes i and j are already connected, all the network nodes which are up to the maximum Euclidean distance of D from i are connected to that node, and all nodes up to D from j are connected to that other node. In case the two nodes i and j are disconnected, all network nodes which are no further than D , respectively to each node i and j , are disconnected from them. In this way, the ‘pros’ and ‘cons’ states in the traditional Sznajd model are respectively associated to *connection* and *disconnection*.

The complex networks obtained by the above described procedure typically exhibit several interconnected patches of highly connected nodes (communities), which is not unlike the distribution of communication and energy in a town or country. Such a distribution of connections and communities provides an interesting prototype for investigating the communication between nodes in the sense suggested in [16], i.e. regarding the communication between nodes at different positions. Of particular interest is the *accessibility* between nodes belonging to distinct communities. While limited information about such topological aspects can be supplied by traditional measurements such as the node degree and clustering coefficient [6, 7], in this work we apply the hierarchical measurements recently introduced in [13, 14, 15]. Such

measurements, which include hierarchical extensions of the node degree and clustering coefficient, consider not only the immediate neighborhood of each network nodes, but all the hierarchical levels defined by taking into account each node as a reference. As shown recently [15], such hierarchical measurements provide a comprehensive characterization of the topology of traditional network models such as random, scale-free (Barabási-Albert –BA [12, 6]) and geographic-regular (i.e. a mesh). The application of such hierarchical measurements to the Sznajd models introduced in this article provides a series of interesting results, including the existence of peaks of connectivity along the hierarchies, which are illustrated and discussed, as well as similarities and dissimilarities with the random and scale-free network models.

This work starts by describing the hierarchical measurements of complex networks and follows by presenting the Sznajd geographical complex network and its characterization in terms of hierarchical topological measurements.

2 Hierarchical Characterization of the Topology of Complex Networks

A non-oriented, non-weighted, complex network (or graph) with N nodes and E edges can be completely represented in terms of its adjacency matrix K , such that $K(i, j) = 1$ indicates the presence of a connection between nodes i and j , while absence of connections are marked by null respective values in that matrix. A complex network is said to be *geographical* in case each of its nodes has a well-specified position in a metric space such as R^2 . An example of geographical complex network is the Voronoi models described in [16], as well as real networks heavily influenced by adjacency constraints, such as transportation systems in the real world.

Complex networks have often been characterized in terms of the average node degree and clustering coefficient [6, 7]. While the degree of a specific network node corresponds to the number of edges attached to that node (observe that the degree of node i can be immediately obtained by summing the entries along the column i , or row, in the respective adjacency matrix), the clustering coefficient can be defined in terms of the following equation

$$cc(i) = \frac{e(i)}{e_T(i)} = 2 \frac{e(i)}{n_1(i)(n_1(i) - 1)} \quad (1)$$

where $e(i)$ is the number of edges among the immediate neighbors of i (connections with that node are not considered), $e_T(i)$ is the maximum possible number of connections between those $n_1(i)$ neighbors. Observe

that $0 \leq cc(i) \leq 1$, with values of zero being achieved for complete absence of connections and one for complete connectivity among the $n_1(i)$ nodes.

Although the averages of node degree and clustering coefficient taken over the whole network of interest provide important quantification of the network connectivity, they are highly degenerated in the sense that an infinite number of complex networks may present the same values for those measurements. The recently introduced concept of hierarchical measurements [13, 14, 15] provide valuable complementary information about the topological properties of a network with respect to its several *hierarchical levels*. Given a reference node i , hierarchies along the remainder of the network can be established by considering the nodes which are at successive exact distances, henceforth represented as d , from the reference node i . The edge distance d between two nodes is henceforth understood to correspond to the number of edges along the shortest path between those two nodes¹. Interestingly, the concepts of node degree and clustering coefficient can be immediately extended to consider each hierarchical level with respect to the reference node, and the respective averages over the network used for a more comprehensive characterization of the connectivity of the studied networks [14, 15].

Let us define the *neighborhood* of a node i at edge distance d as the set of nodes $R_d(i)$ which are exactly at distance d from i , and let $\gamma_d(i)$ be the subnetwork defined by those nodes plus the interconnections inherited from the original complete network. Such a subnetwork (and sometimes the set of nodes $R_d(i)$) have been called the *ring* of radius d centered at the reference node i [14, 15]. The *hierarchical degree* of node i at edge distance d can be defined as the number of edges between the subnetworks $\gamma_d(i)$ and $\gamma_{d+1}(i)$. As such, the hierarchical node degree at distance d ultimately considers the whole network of i up to distance d as being a single enlarged node, whose degree is naturally expressed as above. The *hierarchical clustering coefficient* can be defined in analogous manner in terms of the following expression

$$cc_d(i) = 2 \frac{e_d(i)}{n_d(i)(n_d(i) - 1)}. \quad (2)$$

where $e_d(i)$ corresponds to the number of edges among the nodes in the subnetwork $\gamma_d(i)$ and $n_d(i)$ to the number of nodes in that subnetwork. As with the traditional clustering coefficient, we also have that $0 \leq cc_d(i) \leq 1$, with analogous interpretations.

Other hierarchical measurements which can be used to provide additional information about the connec-

tivity of the studied network include the following:

Hierarchical number of nodes ($n_d(i)$): the number of nodes in the ring $\gamma_d(i)$, which is equal to the cardinality of $R_d(i)$. This measurement has been found to be correlated with the hierarchical node degree, although usually lagged ahead of that measurement. Observe that the hierarchical number of nodes tend to be larger (or at the most equal) to the hierarchical node degree because of the *convergence* of the edges while extending from one ring to the next.

Intra-ring node degree ($A_d(i)$): correlated to the hierarchical node degree, this measurement provides the average degree among the nodes of $\gamma_d(i)$. Observe that only the edges between the nodes in that subnetwork are taken into account.

Inter-ring node degree ($E_d(i)$): corresponds to the average degree of the nodes of $\gamma_d(i)$ considering only the connections extending directly to the next ring $\gamma_{d+1}(i)$. Because the nodes with higher degree (e.g. hubs) are more likely to appear in the first hierarchical levels (they are more likely to be connected to the reference node), this curve tends to decrease more steadily with d in case the network presents many hubs (e.g. scale-free).

Hierarchical common degree ($C_d(i)$): the average of the traditional node degree taken for each successive ring $\gamma_d(i)$. This measurement has been found [15] to provide good discrimination for different network models, presenting a definitely accentuated decrease for scale-free models and an interval of sustained value for random and regular models.

Together with the hierarchical node degree and hierarchical clustering coefficient, a total of 6 hierarchical measurements are therefore obtained which are considered in the current article for the characterization of the Sznajd geographical complex network. Observe that, given the finite size of real networks, *all* such hierarchical measurements tend to zero after some value of d , being typically characterized by one or more peaks at characteristic values of d . An analytical expression for the hierarchical node degree signature for random networks — as well as illustration of the above measurements for random, scale-free and regular network models, have been described in [15]. Such results indicate that the number of hierarchical levels in a network tends to decrease for large average node degree.

Given the patched nature of the complex networks obtained by the herein suggested Sznajd complex model, the above described hierarchical measurements stand out as particularly suitable for the characterization of the intricate connectivity, including bottlenecks, of the patched networks resulting from the Sznajd dynamics over the network edges.

¹It should be observed that there are two distances considered in the current article: the distance d between nodes along network paths, measured in terms of edges; and the Euclidean distance D measured between the positions of nodes in the Euclidean square $L \times L$

3 The Sznajd Geographical Complex Network Model

We now describe how a geographical complex network can have its connections specified by the dynamical evolution of a feedbacked Sznajd model. The geographical network is assumed to be spatially constrained within the square box of side L , while the nodes are uniformly distributed along such space. For the sake of improved visualization, a minimal Euclidean distance D_{min} is observed between any two network nodes, therefore avoiding spatial superposition of nodes. Once the nodes are distributed, a network — henceforth called the *underlying* network — is obtained which includes all connections between any two nodes which are no further apart than a maximum Euclidean distance value D_{max} . The underlying network contains all connections which are possible to be created or eliminated during the subsequent Sznajd dynamics. This network is henceforth represented in terms of its adjacency matrix U , whose respective number of edges is henceforth expressed as N_U .

Figure 1(a) illustrates such a network obtained for $N = 1000$, $L = 500$ and $D_{max} = 35$. Now, the *initial configuration* of the complex network (and Sznajd system) is obtained by taking each of the edges in the potential network with uniform probability p . The initial complex network is henceforth represented by its adjacency matrix I . Figure 1(b) shows such a network obtained from the potential network in (a) by using $p = 0.5$. The evolving network is represented by its adjacency matrix K , with $K = I$ at the initial step.

Now, a feedbacked Sznajd dynamics is imposed on the above specified initial configuration as follows. An edge (i, j) is sampled uniformly from the adjacency matrix U at a time. Observe that such edge may or may not exist in the currently evolving network represented by K . In case it exists (i.e. $K(i, j) = 1$), all immediate neighbors of i and j are identified and connected to those two nodes. In case the edge (i, j) does not exist in the current version of the evolving network (i.e. $K(i, j) = 0$), all immediate neighbors of i and j are disconnected from those two nodes. Observe that such a dynamics corresponds to the traditional Sznajd model where the ‘pros’ and ‘cons’ opinions are now respectively associated to *connected* and *disconnected*.

In case such a dynamics is allowed to take place for a long enough period of time, the resulting network will be either a completely connected or completely disconnected network, of little interest to the complex network community. More interesting connectivity can be obtained by considering the Sznajd contrarian feedback scheme described in [5]. Now, each time an edge is randomly sampled from U and

used to update the network as described above, another edge e taken uniformly from U is subsequently sampled with probability q and the respective edge in K receives the value contrary to the current dominant ‘opinion’ in the network. For instance, in case the current network represented by K has more than $N_p/2$ edges, the edge e is disconnected in K .

After stabilization, which can be identified by observing the overall mean magnetization to remain limited around zero during a pre-specified interval of time, geographical complex networks are obtained which are characterized by spatially delimited patches of highly connected communities which are themselves less intensely interconnected one another. Figure 1(c) shows such a network obtained from the initial condition in (b) by considering $q = 0.4$. Such networks are remindful of those resulting from the distribution of energy and communication cables, or even social relationships, in a town (where each community would correspond to a building or institution) or a densely populated country (the communities corresponding to the towns). Because of such spatial distribution of connections, the obtained networks are particularly interesting to be used as prototypes for studies of communication and accessibility between nodes in the sense discussed, for instance, in [16]. Of particular interest is the number of hierarchies taken for a specific node to broadcast along the network, as well as the identification of the many bottlenecks. Because of the patched nature of the connections, it is expected that surges of communication will take place every time a new highly connected community is reached.

We show in the next section how the set of hierarchical measurements of network topology recently introduced in [13, 14, 15] can be used in order to obtain a comprehensive characterization of intricate topology of Sznajd geographical complex networks.

4 Simulation Results

The above obtained Sznajd complex network model was analyzed by using the set of hierarchical measurements reviewed in Section 2. Simulated random and BA network models with the same number of nodes (1000) and similar connectivity ($\langle k \rangle \approx 5.5$) were also characterized by the same measurements for the sake of comparison. We divide the hierarchical characterization of the networks into two subsections, one considering the average \pm standard deviation of the measurements considering all network nodes, and another taking into account the hierarchical measurements at individual node level.

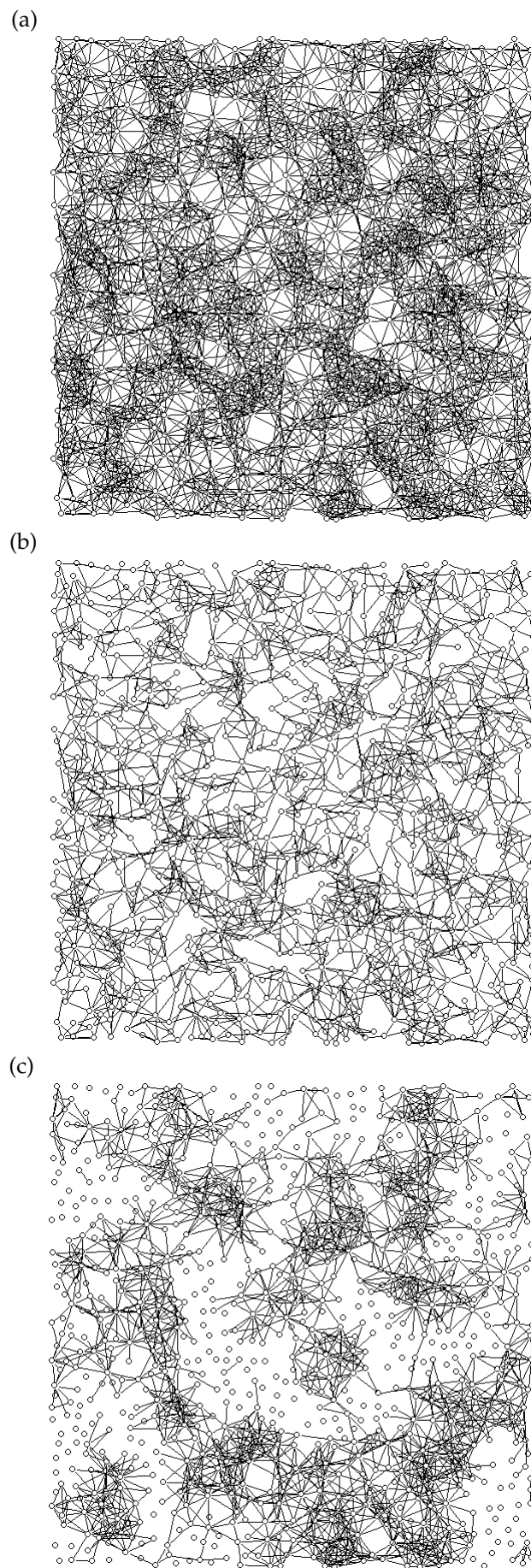


Figure 1: Example of underlying network (a), a respectively derived initial configuration (b) and complex network obtained by using the feedback Sznajd dynamics (c).

4.1 Global Features

Figure 2 shows the average \pm standard-deviations of the set of hierarchical measurements obtained for the Sznajd geographical network model (first column) as well as simulated random and BA models considering the same number of nodes (1000) and similar connectivity (5.5). A marked difference is observed between the Sznajd and the other models. First, it includes many more hierarchical levels, up to about 30 levels, while the number of hierarchies in the other models is limited to about 10. As expected, the shape of the curves obtained for the hierarchical number of nodes, hierarchical node degree and hierarchical common degree involve three distinct regions along the distance values d : (i) an increase corresponding to the expansion of the neighborhood; (ii) a peak; and (iii) a decrease implied by the limited size of the network. When observed comparatively one another, the shape of the curves obtained for the hierarchical number of nodes and hierarchical node degree for the three cases resulted similar, except for the respective heights and widths. The other measurements were characterized by similarities and dissimilarities. More specifically, the inter-ring degree and hierarchical clustering coefficient of the Sznajd model (see Figure 2(c) and (f), respectively) resembled — in shape, not absolute values — the respective measurements obtained for the BA model, as indicated in Figure 2(c) and (i). At the same time, the hierarchical common degree of the curves obtained for the Sznajd and random models resulted similar in shape and absolute values. The intra-ring degree of the former model was not similar to any of the other two models. Generally speaking, the Sznajd network was characterized by sustained intra-ring degree (see Figure 2(d)) along the initial distance values (roughly between 1 and 15), which suggests that the rings obtained for that model have similar interconnectivity. A sustained interval was also observed for the hierarchical common degree (Figure 2(e)) and hierarchical clustering coefficient (Figure 2(f)) obtained for the Sznajd network. Such sustained behavior are similar to those observed in [15] for regular networks, suggesting that the Sznajd model shares topological features. At the same time, the nearly constant values of hierarchical node degree obtained for the Sznajd model (Figure 2(d)) indicates that nodes with similar traditional degrees are incorporated along about half of the hierarchical levels, reflecting the absence of hubs in the Sznajd model. However the hierarchical clustering coefficient of the Sznajd case was different in the sense that it contained a marked peak at $d = 22$ (see Figure 2(f)), which is more similar to corresponding measurements obtained for the scale-free models.

4.2 Individual Features

We now turn our attention to the analysis of the topological features of the three considered models at an *individual* level. More specifically, we concentrate our attention on the leftmost upper network node, namely that indicated by the arrow in Figure 3. This figure corresponds to the same network in Figure 1(c), except that the edges between the communities **A** and **B** were deleted in the network shown in Figure 3, which was done in order to provide a comparative context.

Figure 4 shows the hierarchical measurements obtained for the Sznajd network in Figure 1(c) (first column in Figure 4) and the modified network in Figure 3 (second column in Figure 4). Several interesting features can be inferred from such results. First, observe that all measurements obtained for the modified Sznajd network extend along longer values of d , which is a consequence of the fact that the original bypass from community **A**, to which the reference node (arrow) belongs, to community **B** has been removed. This small modification implied that the increasing hierarchies had to go through the rest of the modified network before reaching the community **B**. Actually, the peaks at the left and right hand sides of the curve in Figure 4(g) — i.e. the portions of that curve extending from $d = 1$ to 11 and from $d = 29$ to 35, respectively — have been verified to correspond to the nodes in communities **A** and **B**, respectively.

Another interesting feature is the similarity, except for a small shift, between the curves obtained for the hierarchical number of nodes and the hierarchical node degrees observed for both networks. It has been verified that the valleys (i.e. minimum relative peaks) along any of these curves correspond to the *bottlenecks* existing between the several spatially distributed network communities. Therefore, the identification of such minimum peaks presents good potential for community finding in such networks, as well as for any other network model. The curves of the inter and intra-ring degrees and hierarchical common degrees obtained for the Sznajd network in Figure 1(c) were characterized by approximately sustained values, reflecting the degree regularity of that model. The counterpart curves obtained for the modified Sznajd network (second column in Figure 4) were characterized by being less regular, providing an illustration of the criticality of small network changes upon the respective topological features. Finally, the hierarchical clustering coefficients results similar for both cases.

5 Conclusions

Unlike previous works [19], which ran Sznajd dynamics on complex networks, we adopted the complementary approach and used the Sznajd dynamics to produce complex networks. The first point to be observed

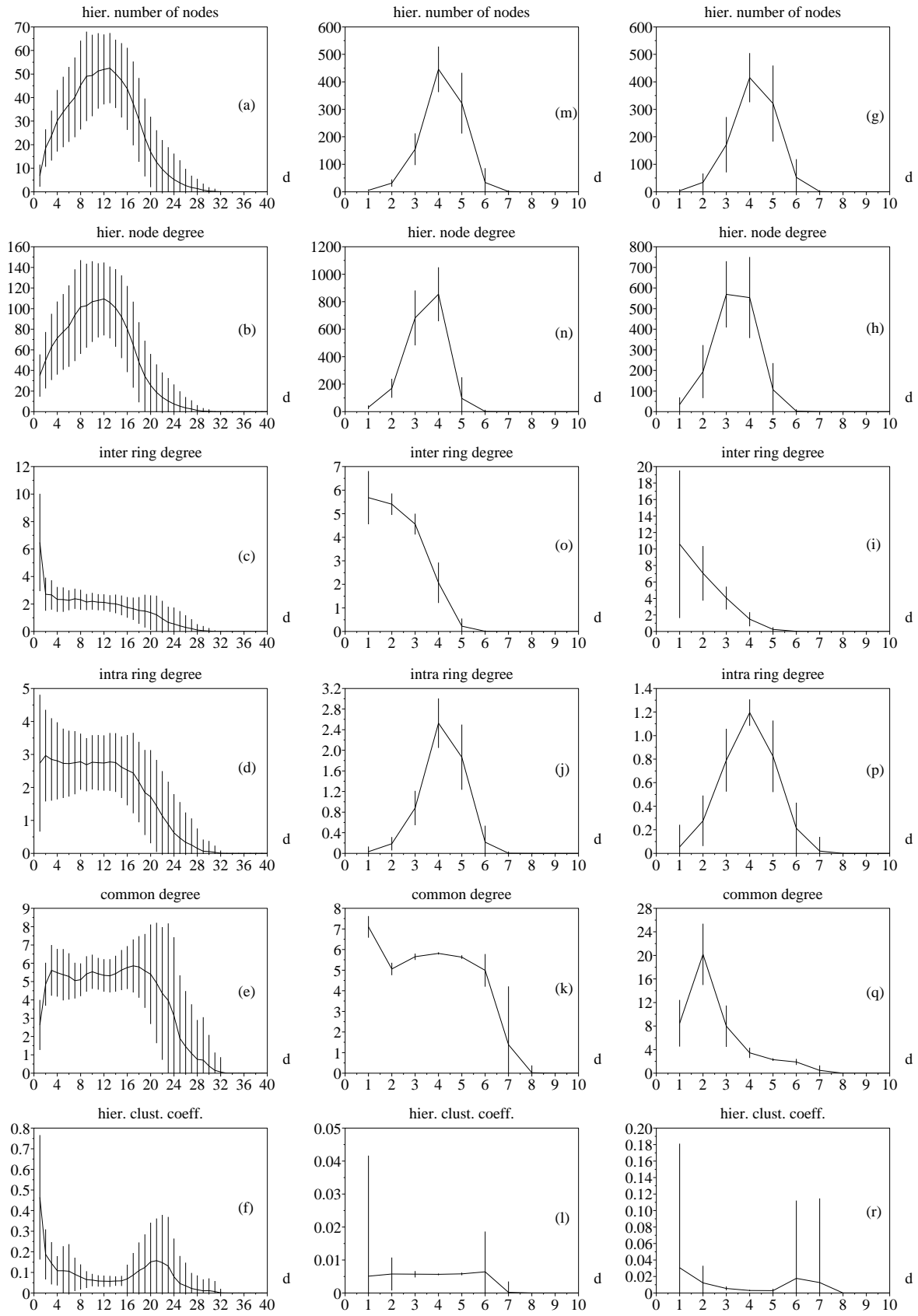


Figure 2: Hierarchical measurements obtained for the feedbacked Sznajd mode (first column) and random (second column) and BA (third column) simulated networks considering the same number of nodes and similar connectivity.

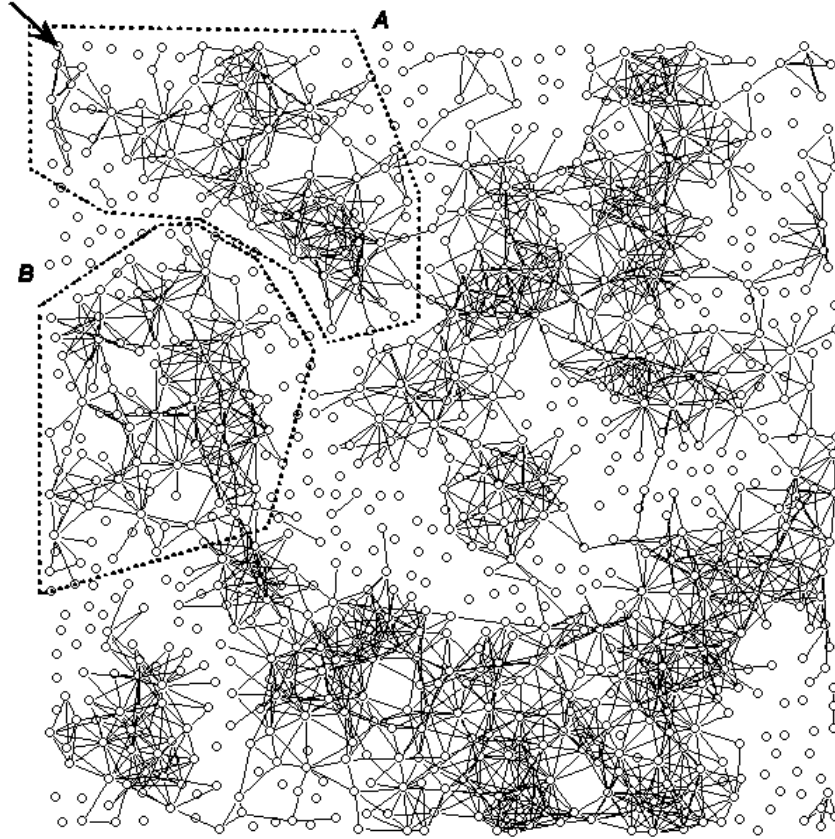


Figure 3: A modified version of the Sznajd network in Figure 1(c), where the edges connecting the communities marked as **A** and **B** have been deleted for the sake of comparisons. The network node considered for the individual hierarchical analysis is indicated by the arrow at the upper, rightmost corner of the network

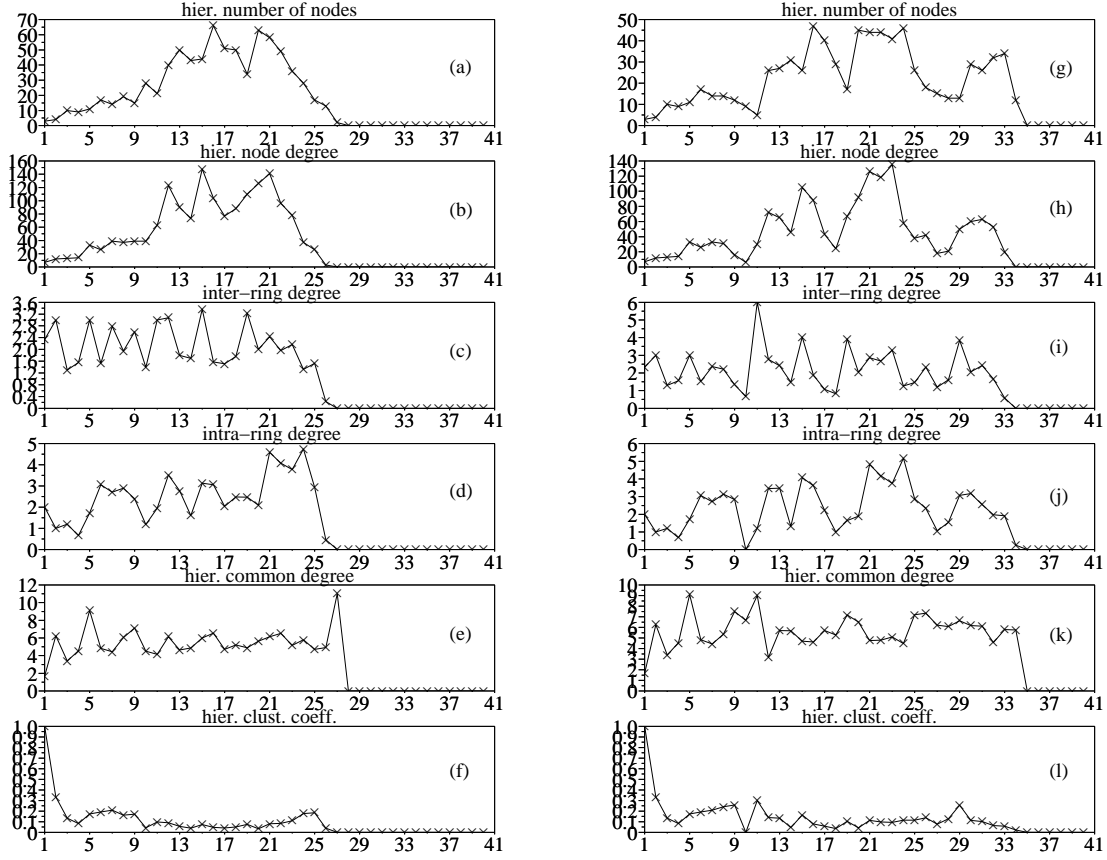


Figure 4: Hierarchical measurements obtained for the feedbacked Sznajd mode (first column) and random (second column) and BA (third column) simulated networks considering the same number of nodes and similar connectivity.

is that the association between ‘pros’ and ‘cons’ with the *connected* and *disconnected* states of edges, respectively, can be used to define a broad Sznajd-related family of complex networks. In other words, the Sznajd dynamics is performed over the network possible edges.

In the current work we explored the particular Sznajd dynamics where randomly selected edges are assigned the opposite of the current dominant connectivity among the evolving network. The obtained geographical networks were characterized by spatial patches of high connectivity (i.e. communities) which were less intensely connected one another. Such models, which are remindful of distribution of intercommunication, energy or even social contacts in real structures such as towns and countries, provide an interesting prototype for studying community and connectivity in complex networks.

Several interesting topological properties of the Sznajd networks were identified by using a recently introduced set of hierarchical measurements including hierarchical extensions of the traditional node degree and clustering coefficient. The analysis of the Sznajd model was performed comparatively with random and scale-free (BA) models simulated with the same number of nodes and similar connectivity and by considering global measurements (i.e. average and standard deviation) involving all network nodes as well as at the individual node measurement level. A series of interesting results was obtained, including the identification of similarities and dissimilarities of specific measurements between the Sznajd and random/BA models. Generally, the Sznajd model was characterized by sustained values of inter and intra-ring degrees, as well as hierarchical common degree and hierarchical clustering coefficient. The latter measurement also resulted similar to that obtained for the BA case. The Sznajd model was globally characterized as exhibiting high node degree regularity, a consequence of the spatial adjacencies underlying that model. The individual analysis of the hierarchical features was performed considering a modified version of the Sznajd model, which lead to major effects over the respective measurements. Such examples clearly illustrated the potential of the hierarchical measurements for identification of communities and connectivity bottlenecks.

The above reported developments pave the way to a series of future developments, including alternative growth network schemes and applications. For instance, it would be interesting to consider Sznajd dynamics involving long-range connections [18] as well as higher spatial dimensions [19]. Other possibilities involve the use of Sznajd geographical networks as models of biological pattern formation, including neuronal modules/systems and gene expression and cell signaling. It would also be interesting to develop a more systematic investigation of the potential of the

hierarchical measurements for identification of communities in complex networks.

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